6.1 Radian Measure

A Angles

Angle is the region between *two rays* (called *arms*) starting from the same end-point called the *vertex* of the angle.
One line split the plan in two angles called *straight* angles.
Two perpendicular lines split the plan in four angles called *right* angles.
Any angle less than a right angle is called *acute* angle.
Any angle greater than a right angle and less than a straight angle is called *obtuse* angle.
An angle that is greater than a straight angle is called *reflex* angle.
If the arms are coincident then a *full turn* angle is formed.

B Sign

If the rotation of the initial arm toward the terminal arm is a *counter clockwise* rotation the angle is considered *positive*.
Otherwise the angle is *negative*.

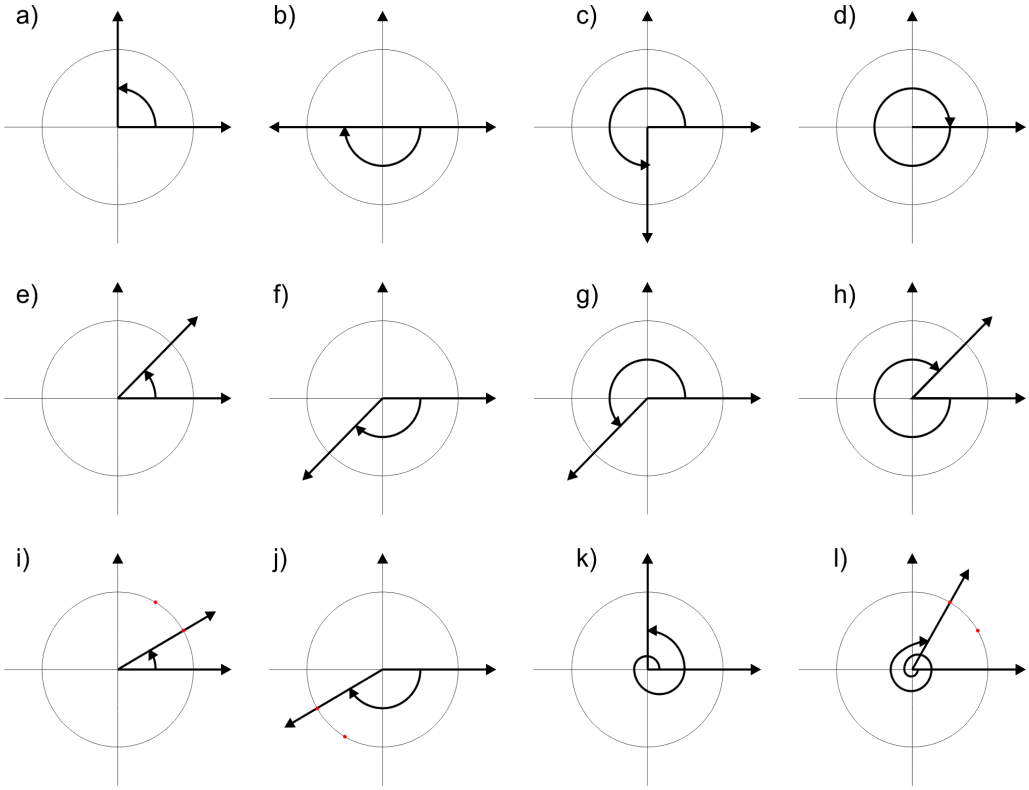
C Standard Position

If the *initial arm* is *parallel to the x-axis* then the angle is in standard position.

D Degrees

The measurement of a *right angle* in degrees is (by convention) $90°$.

Ex 1. Find the measurement of each angle (given in standard position) in degrees.



E Radians

Angles can be measured also in *radians*.
The measure of an angle in radians is given by the ratio between the *arc length* subtended by the angle and the *radius length* of than circle:

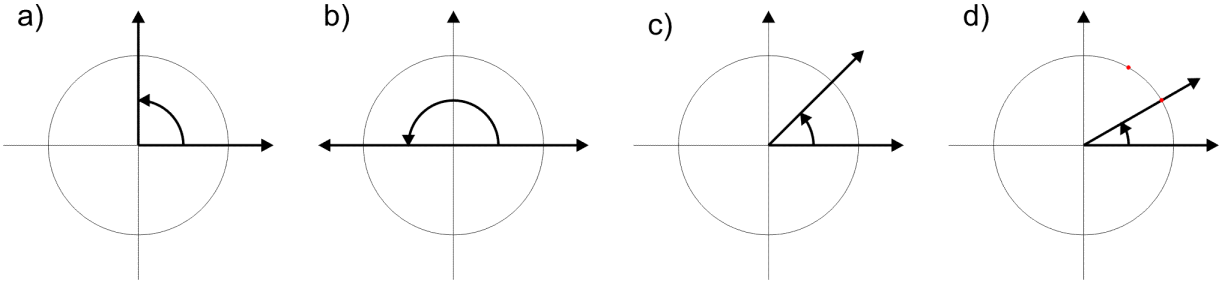
$$\alpha = \frac{s}{R}$$

Note. To calculate the angles in radians recall the *circumference* length formula:

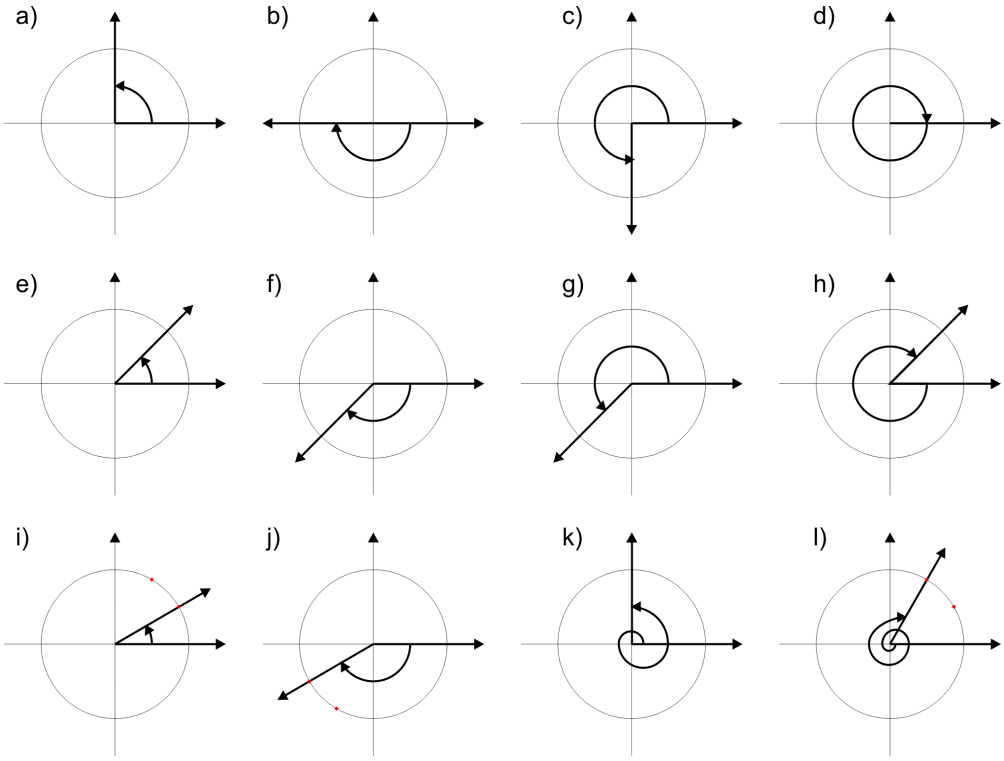
$$C = 2\pi R$$

Ex 2. Use the definition to find the measurement of each angle in radians.

a) b) c) d)



Ex 3. Find the measurement of each angle in radians.



D Conversion formula

The *conversion* between degree and radian measure is based on the equality:

$$180° = \pi \ radians$$

which is equivalent to:

$$1° = \frac{\pi \ radians}{180} \quad \text{or}$$

$$1 \, radian = \frac{180°}{\pi}$$

Ex 4. Do the required conversions.

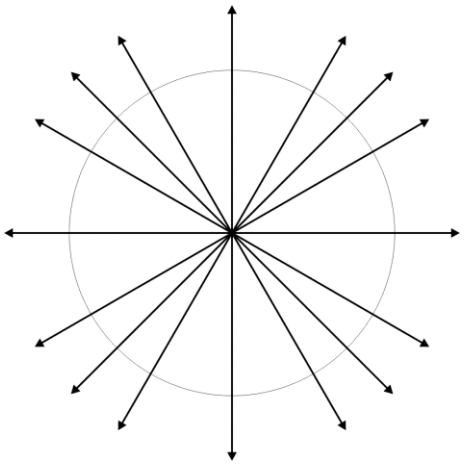
a) $10° = ? \, radians$

b) $-1° = ? \, radians$

c) $\sqrt{2} \ radians = ?°$

d) $-0.1 \, radians = ?°$

Ex 5. Complete the diagram with the angles expressed in degrees and radians.



E Arc Length

The arc length subtended by the angle θ (in radians) on a circle of radius R is given by:

$$s = \theta R$$

Ex 6. Find the length of the arc subtended by an angle of $50°$ on a circle of radius $30cm$.

F Sector Area

The sector area delimited by an angle θ (in radians) and a circle of radius R is given by:

$$A = \frac{1}{2}\theta R^2$$

Ex 7. Find the area delimited by an angle of $120°$ and a circle of radius $10cm$.

Reading: Nelson Textbook, Pages 316-320
Homework: Nelson Textbook, Page 321: #5, 6, 7abc, 8abc, 11, 16

